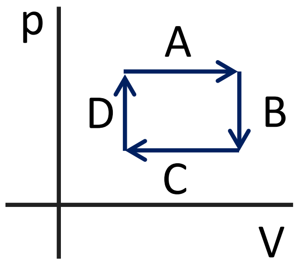
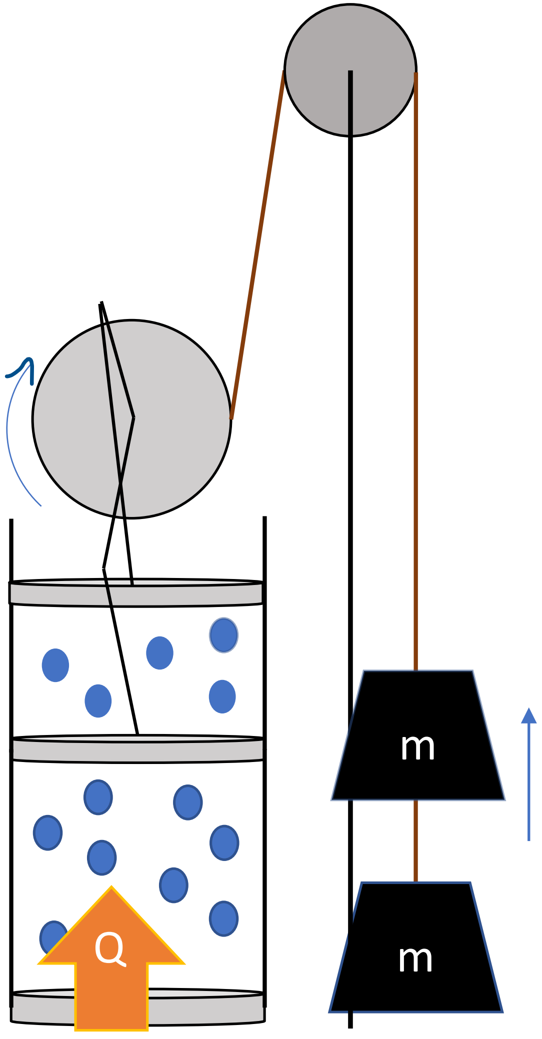
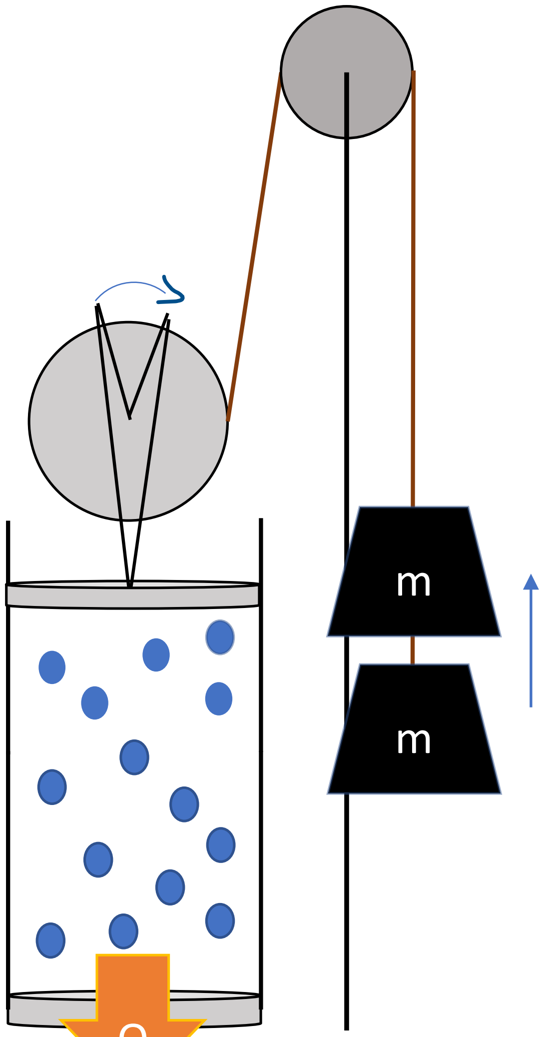
F.8 Engines

Let's consider a typical engine process…

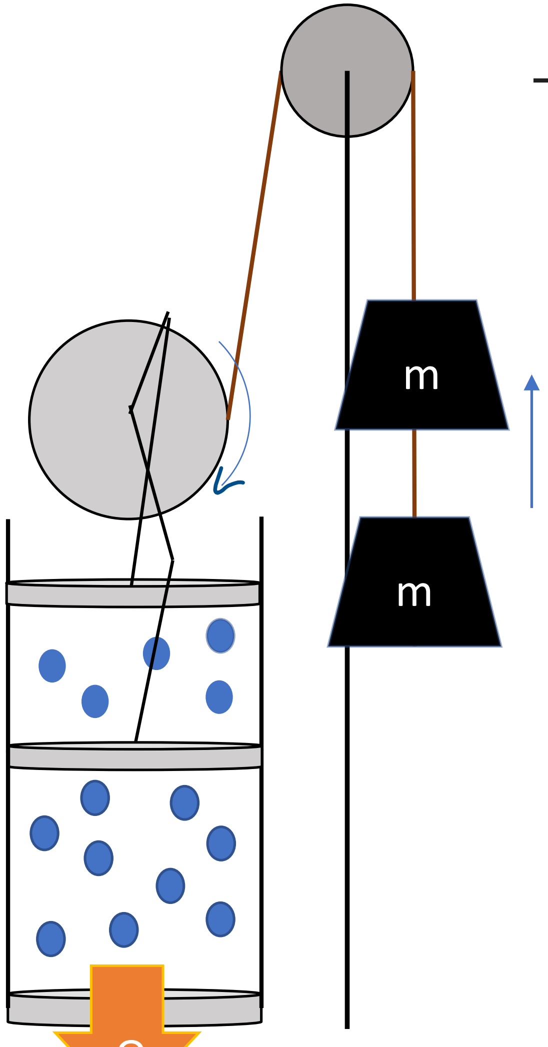




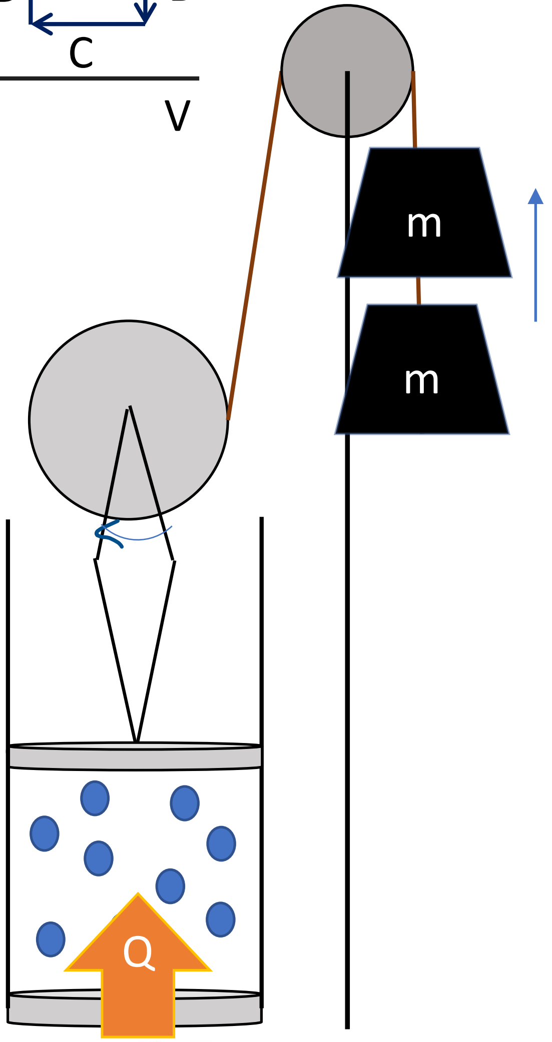
A. heat isobarically, driving piston upwards



B. cool isometrically to prepare for piston's descent



C. cool isobarically, allowing piston to descend



D. heat isometrically to prepare for piston's ascent

F.8 Engines

Now we'd like to analyze the engine cycle in more detail….

First, it's common to draw in the isotherms representing the temperatures of the hot and cold sources:

And then we'd like to calculate….

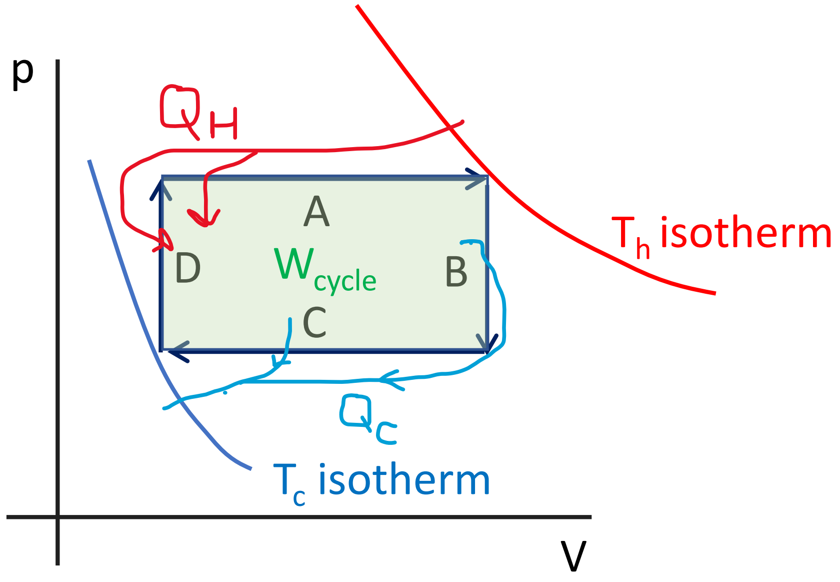
The heat added to the gas by the hot temperature source, Q_H, during the cycle.

The heat subtracted by the cold temperature source, Q_C, during the cycle.

The work done on the piston by the gas, during the cycle: W_{cycle} = area inside curve.

The efficiency of the engine, η:

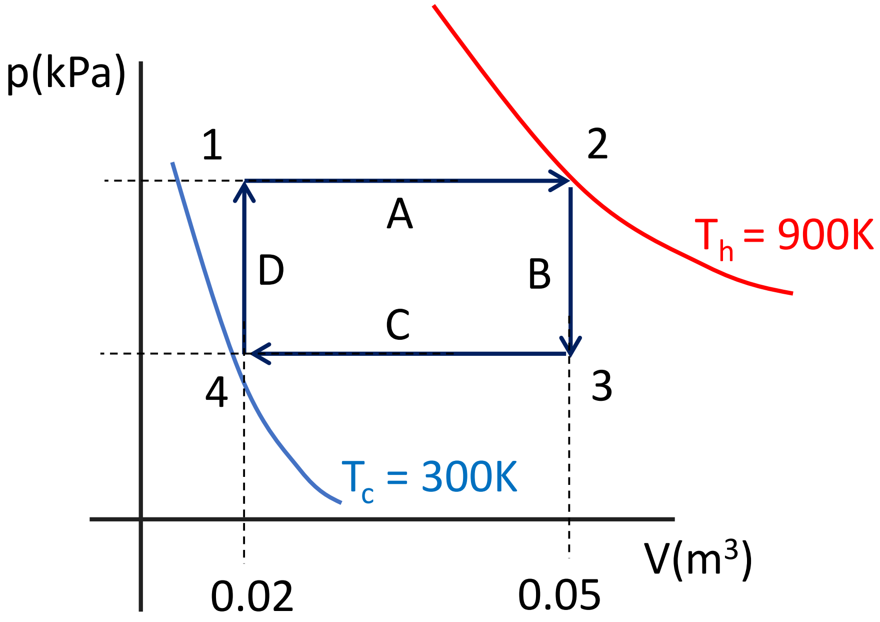
$$\eta = \frac{\text{energy you get}}{\text{energy you have to put in}} = \frac{W_{cycle}}{Q_H}$$



Note an important relationship between W_{cycle}, Q_H, Q_C, obtained from the first law:

$$-W_{gas(cycle)} + Q_{cycle} = \Delta E_{cycle} \longrightarrow -W_{cycle} + (Q_H - Q_C) = 0$$

$$\longrightarrow \boxed{W_{cycle} = Q_H - Q_C}$$



F.8 Engines

Let's say we have n = 1.2 mol of diatomic gas operating in an isometric/baric cycle between 0.02m^3 and 0.05m^3, and between reservoirs with temperature T_c = 300K and 900K respectively.

What are the pressures p_1, p_4, and the temperatures T_1, T_3?

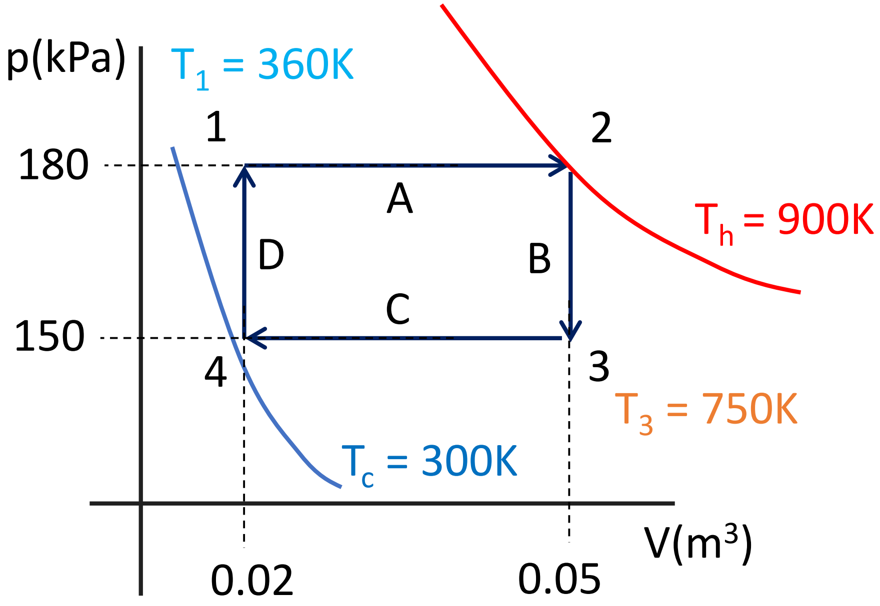
$$p_4 = \frac{nRT_4}{V_4} = \frac{(1.2)(8.31)(300)}{0.02} = 1.5\times10^5\,\text{Pa} \quad = 150\text{kPa}$$

$$p_1 = \frac{nRT_1}{V_1} = \frac{nRT_2}{V_2} = \frac{(1.2)(8.31)(900)}{0.05} = 1.8\times10^5\,\text{Pa} \quad = 180\text{kPa}$$

$$T_1 = \frac{p_1V_1}{nR} = \frac{(1.8\times10^5)(0.02)}{(1.2)(8.31)} = 360\text{K}$$

$$T_3 = \frac{p_3V_3}{nR} = \frac{(1.5\times10^5)(0.05)}{(1.2)(8.31)} = 750\text{K}$$

F.8 Engines



What's W_A, Q_A,?

$$W_A = p\Delta V$$
$$= (180\times10^3\,\mathrm{Pa})(0.05\,\mathrm{m}^3 - 0.02\,\mathrm{m}^3)$$
$$= 5400\,\mathrm{J}$$

$$Q_A = W_A + \Delta E_A$$
$$= W_A + \frac{f}{2} nR\Delta T_A$$
$$= 5400 + \frac{5}{2}(1.2)(8.31)(900-360)$$
$$= 18900\,\mathrm{J}$$

What's W_B, Q_B?

$$W_B = 0$$

$$Q_B = W_B + \Delta E_B$$
$$= W_B + \frac{f}{2} nR\Delta T_B$$
$$= 0 + \frac{5}{2}(1.2)(8.31)(750-900)$$
$$= -3700\,\mathrm{J}$$

What's W_C, Q_C?

$$W_C = p\Delta V$$
$$= (150\times10^3\,\mathrm{Pa})(0.02\,\mathrm{m}^3 - 0.05\,\mathrm{m}^3)$$
$$= -4500\,\mathrm{J}$$

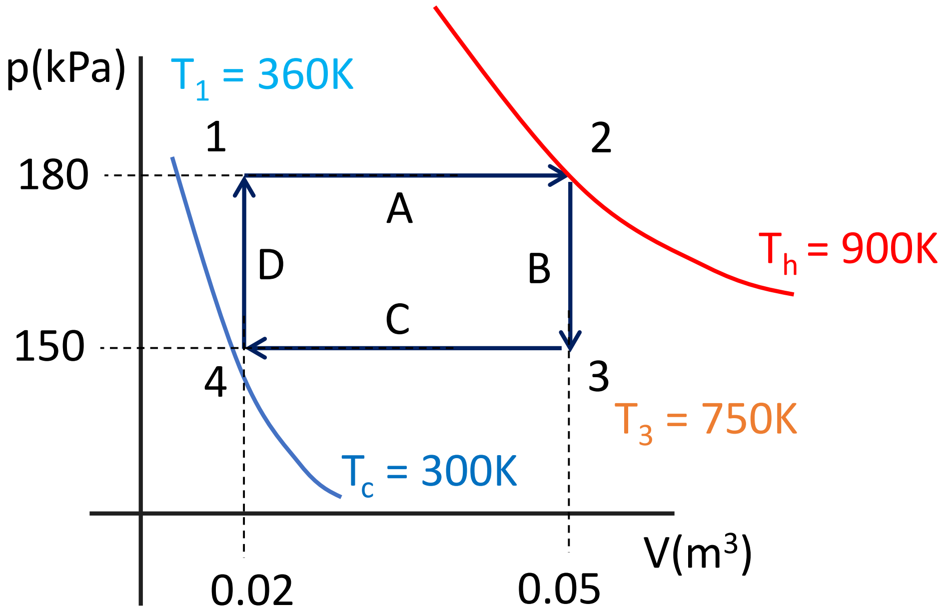
$$Q_C = W_C + \Delta E_C$$
$$= W_C + \frac{f}{2} nR\Delta T_C$$
$$= -4500 + \frac{5}{2}(1.2)(8.31)(300-750)$$
$$= -15800\,\mathrm{J}$$

What's W_D, Q_D?

$$W_D = 0$$

$$Q_D = W_D + \Delta E_D$$
$$= W_D + \frac{f}{2} nR\Delta T_D$$
$$= 0 + \frac{5}{2}(1.2)(8.31)(360-300)$$
$$= 1500\,\mathrm{J}$$

F.8 Engines



What is the work done by the gas during cycle?

$$W_{cycle} = W_A + W_B + W_C + W_D$$

$$= 5400\text{J} + 0\text{J} - 4500\text{J} + 0\text{J}$$

$$= 900\text{J}$$

What is the efficiency of the engine?

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{900\text{J}}{20400\text{J}} = 0.044 = 4.4\%$$

What's the heat supplied to the gas during cycle?

$$Q_H = \sum +Q's$$

$$= 18900\text{J} + 1500\text{J}$$

$$= 20400\text{J}$$

What's the heat taken from the gas during cycle?

$$Q_C = \left|\sum -Q's\right|$$

$$= \left|-3700\text{J} - 15800\text{J}\right|$$

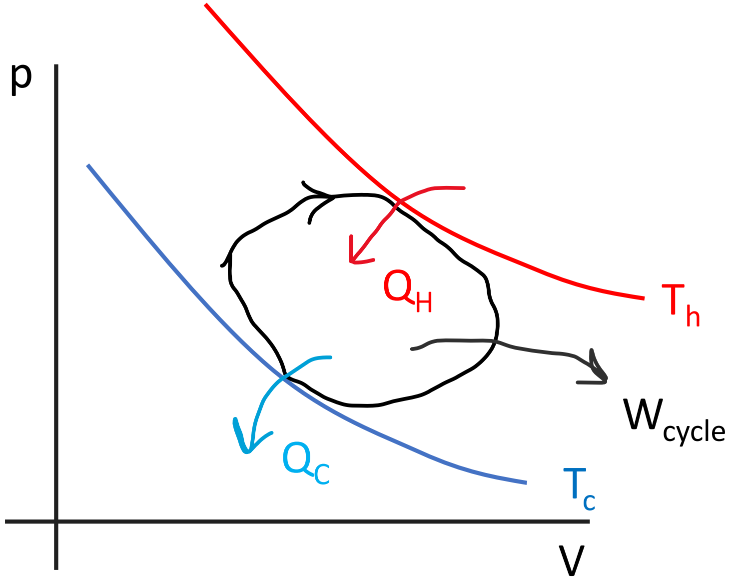
$$= 19500\text{J}$$

How much heat must be supplied to raise 500kg beam 30m into air?

If the engine runs at 2 cycles/second, how long would this take?

$$\eta = \frac{W_{total}}{Q_{H(total)}} \longrightarrow Q_{H(total)} = \frac{W_{total}}{\eta} = \frac{mgh}{\eta} = \frac{(500)(9.8)(30)}{0.044} = 3.3\times10^6\,\text{J}$$

$$\Delta t = (\#\text{cycles})\left(\frac{1\text{s}}{2\text{cycles}}\right) = \left(\frac{W_{total}}{W_{cycle}}\right)\left(\frac{1\text{s}}{2\text{cycles}}\right) = \left(\frac{(500)(9.8)(30)}{900}\right)\left(\frac{1}{2}\right) = 81\text{s}$$



F.8 Engines

Now let's consider why the efficiency was small, and what could be done to augment it. To that end consider an arbitrary cycle and thermal reservoirs.

Generally speaking, heat Q_H, is extracted from the hot reservoir, into the gas:

The gas outputs work, W_{cycle},

Then heat Q_C, is extracted from the gas into the cold reservoir:

The only laws that apply to *any* cycle are the 1st and 2nd law. So what does the 1st law say about our cycle? Applying it to the engine, we already concluded:

$$Q_H - Q_C = W_{cycle}$$

What does the 2nd law say about our cycle? Let's apply it to both the engine and the *reservoirs*:

$$\int \frac{dQ}{T} + \Delta S_{\text{int.}} = \Delta S_{gas} + \Delta S_{hot\,reservoir} + \Delta S_{cold\ reservoir}$$

$$0 + \Delta S_{\text{int.}} = 0 + \frac{-Q_H}{T_H} + \frac{Q_C}{T_C}$$

$$\frac{Q_C}{T_C} - \frac{Q_H}{T_H} = \Delta S_{\text{int.}}$$

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$$Q_H - Q_C = W_{cycle}$$
$$\frac{Q_C}{T_C} - \frac{Q_H}{T_H} = \Delta S_{\text{int.}}$$

Now let's see what implications those two equations have for the efficiency η = W_{cycle}/Q_H.
We'll use the equations to solve for Q_H, and plug it back into η.

$$Q_H - Q_C = W_{cycle}$$

$$+$$

$$T_C\left(\frac{Q_C}{T_C} - \frac{Q_H}{T_H} = \Delta S_{\text{int.}}\right)$$

$$Q_H - \frac{T_C}{T_H} Q_H = W_{cycle} + T_C \Delta S_{\text{int.}}$$

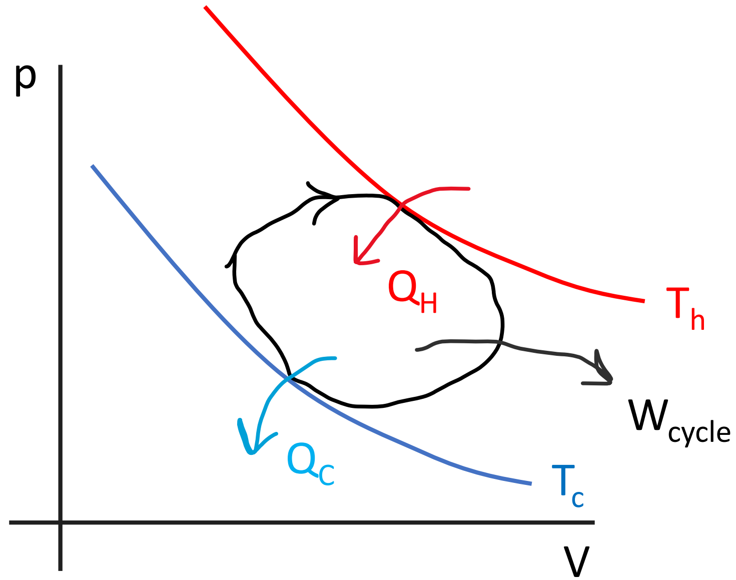
$$\longrightarrow \left(1 - \frac{T_C}{T_H}\right) Q_H = W_{cycle} + T_C \Delta S_{\text{int.}} \longrightarrow Q_H = \frac{W_{cycle} + T_C \Delta S_{\text{int.}}}{1 - \frac{T_C}{T_H}}$$

Plugging into the efficiency $\longrightarrow$ $\eta = \dfrac{W_{cycle}}{\dfrac{W_{cycle} + T_C \Delta S_{\text{int.}}}{1 - T_C/T_H}}$ $\longrightarrow$ $\eta = \left(1 - \dfrac{T_C}{T_H}\right)\dfrac{1}{1 + T_C \Delta S_{\text{int.}}/W_{cycle}}$

This implies the efficiency is maximized when $\Delta S_{\text{int.}} = 0$, in which case we get: $\boxed{\eta_{\max} = 1 - \dfrac{T_C}{T_H}}$

But what would such a cycle look like?

F.8 Engines

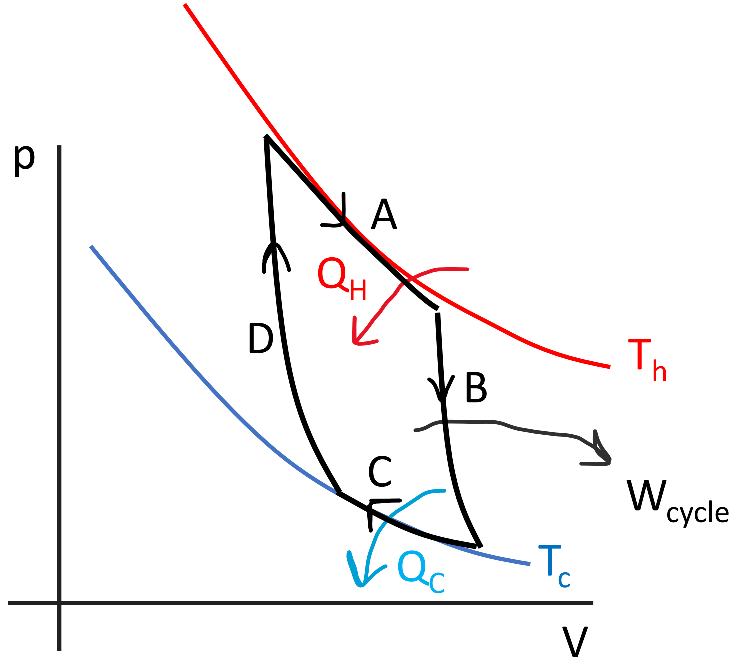


There is only one process that strictly satisfies $\Delta S_{int.} = 0$.

- Isentropic process

But we cannot power an engine with only isentropic processes, because then we could never add heat/energy. But there *is* another process which can come *close* to satisfying $\Delta S_{int.} = 0$. Heat transfer between two objects at unequal temperatures is an *internal* process which *always* increases S, but if the two objects are at close to the same temperature, then the S increase would be small. So what we want is a process where the temperature of the gas remains very close to the temperatures of the reservoirs. So we need an …

- Isothermal process *if* the gas is always held close to the same temperature as the reservoir(s).



So the max efficiency cycle, called the **Carnot** cycle, looks like this:

A. isothermal expansion

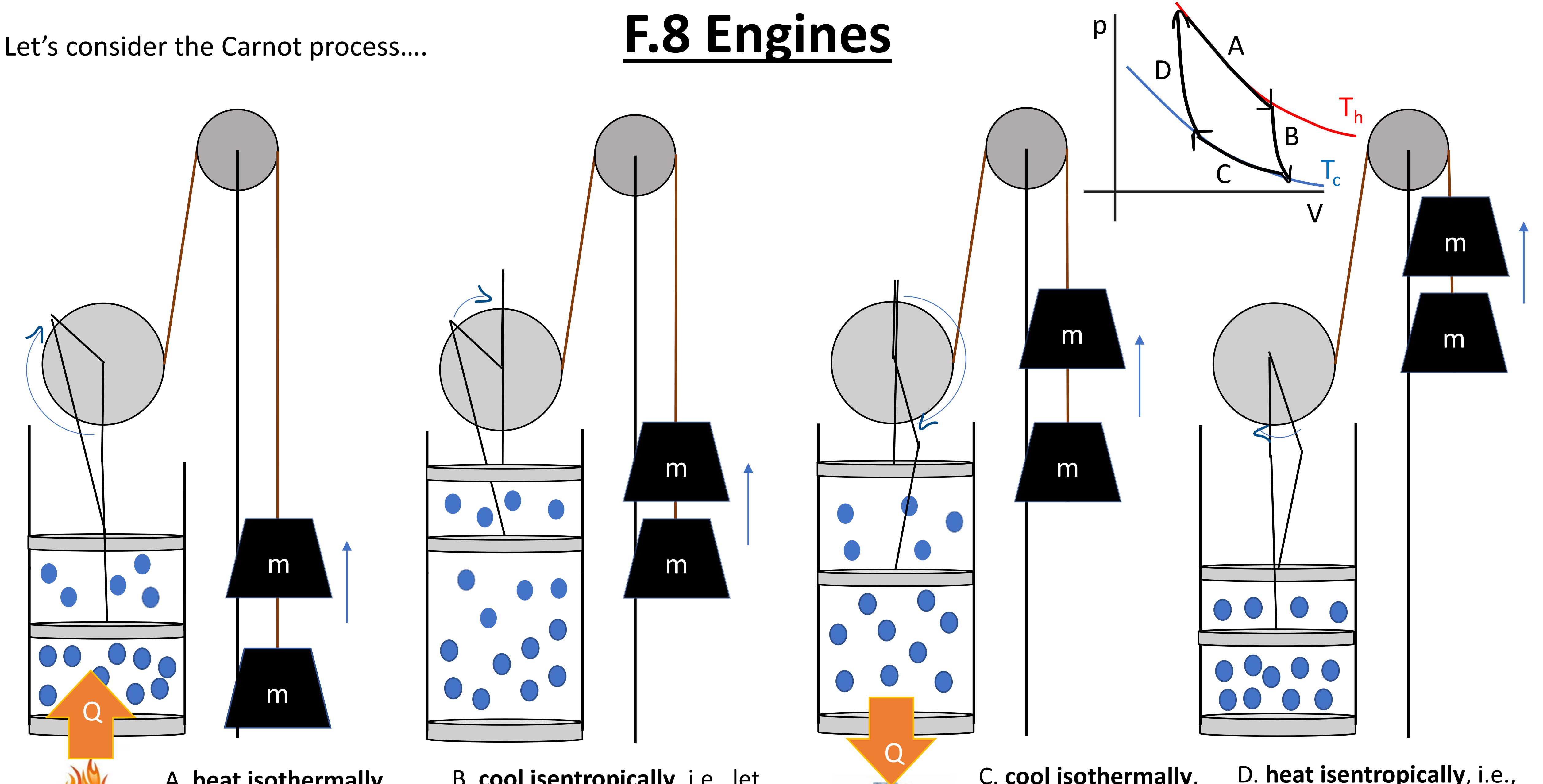
B. isentropic cooling

C. isothermal cooling

D. isentropic heating

F.8 Engines

Let's consider the Carnot process….



A. **heat isothermally**, driving piston upwards

B. **cool isentropically**, i.e., let the gas cool itself by doing work pushing the piston up.

C. **cool isothermally**, allowing piston to descend

D. **heat isentropically**, i.e., let the falling piston do work on the gas, heating it up.

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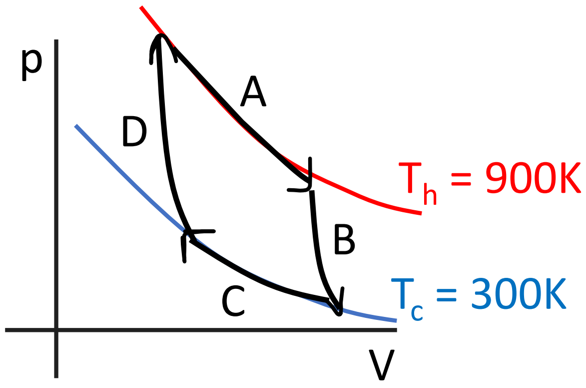

Let's consider a Carnot engine operating between the same two hot/cold reservoirs, as our last engine. What is its efficiency?

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{900} = 0.67 = 67\%$$

How much heat must be supplied to raise 500kg beam 30m into air?

$$\eta = \frac{W_{total}}{Q_{H(total)}} \longrightarrow Q_{H(total)} = \frac{W_{total}}{\eta} = \frac{mgh}{\eta} = \frac{(500)(9.8)(30)}{0.67} = 219\text{kJ}$$

as compared to 3300kJ, for the previous engine cycle.